

Additive Synthesis of a Hard-Synced Square Wave Using Sawtooth Waves

(ノコギリ波を用いた、ハードシンクされた矩形波の加算合成)

Jun. 1, 2026

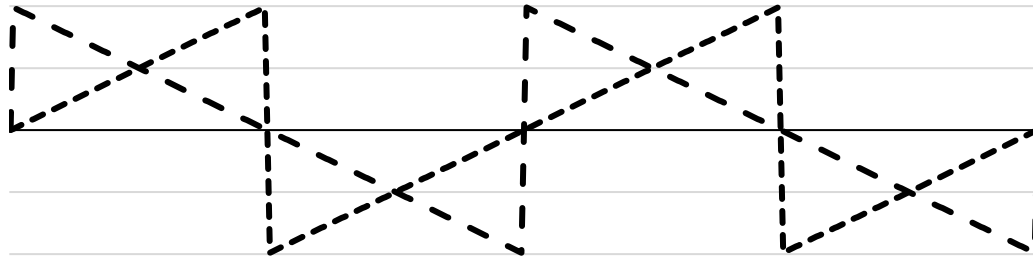
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ISGK Instruments

<https://risgk.github.io/>



1. Additive Synthesis of a Square Wave

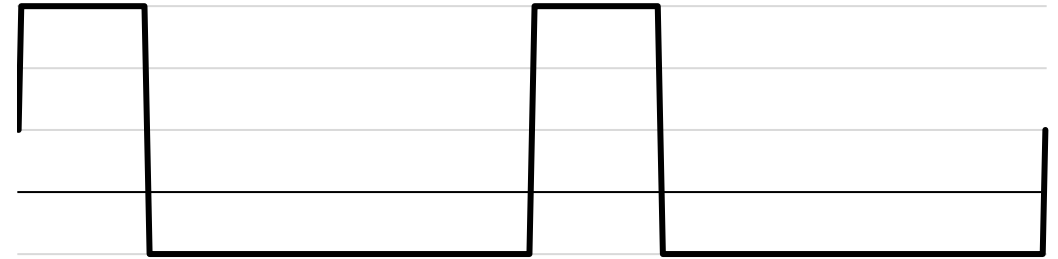


-- Reverse Sawtooth Wave --- Sawtooth Wave

— Square Wave

- By adding a reverse sawtooth wave to a sawtooth wave, a square wave can be synthesized [1]
- When the input sawtooth waves are band-limited, the output square wave remains band-limited (and thus aliasing-free)

2. Additive Synthesis of a Pulse Wave

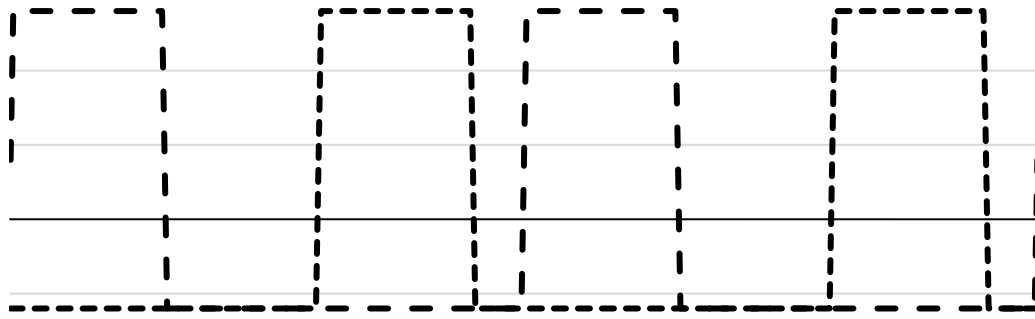


- - Reverse Sawtooth Wave - - - Sawtooth Wave

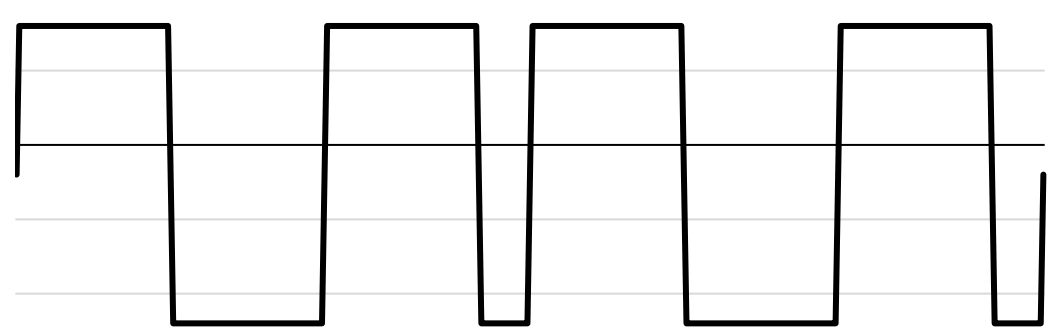
— Pulse Wave

- By adding a reverse sawtooth wave to a sawtooth wave, a pulse wave can be synthesized [1]
- By generating these sawtooth waves using wave tables or PolyBLEP, a pulse wave can be synthesized with any duty cycle and initial phase
- While PWM can be achieved by modulating the phase of the sawtooth wave, measures such as limiting the modulation rate are necessary to prevent noise

3. Additive Synthesis of a Hard-Synced Square Wave



-- Pulse Wave 1 --- Pulse Wave 2



— Hard-Synced Square Wave

- **Result of previous research:** By adding sawtooth waves, a hard-synced sawtooth wave can be synthesized [2]
- **Key result:** By adding pulse waves or sawtooth waves, a hard-synced square wave, a pulse wave, and arbitrary binary-state waveforms can be synthesized
- **This method is efficient:** For instance, if the square wave is 3 octaves higher, only 16 sawtooth waves are required

4. Implementation Example: PRA32-U2

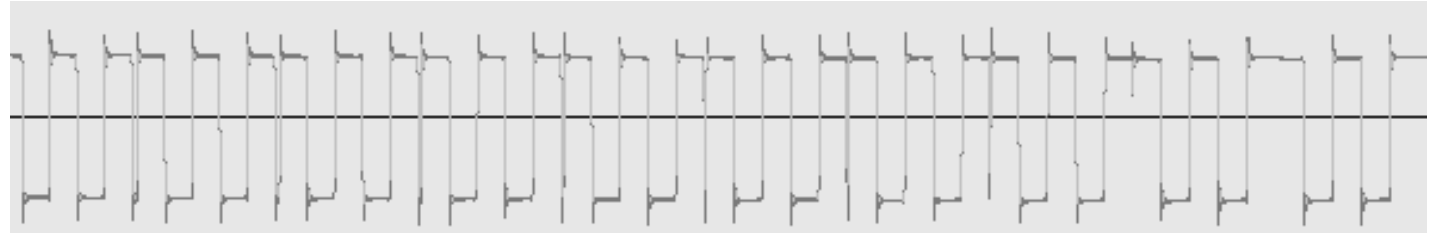
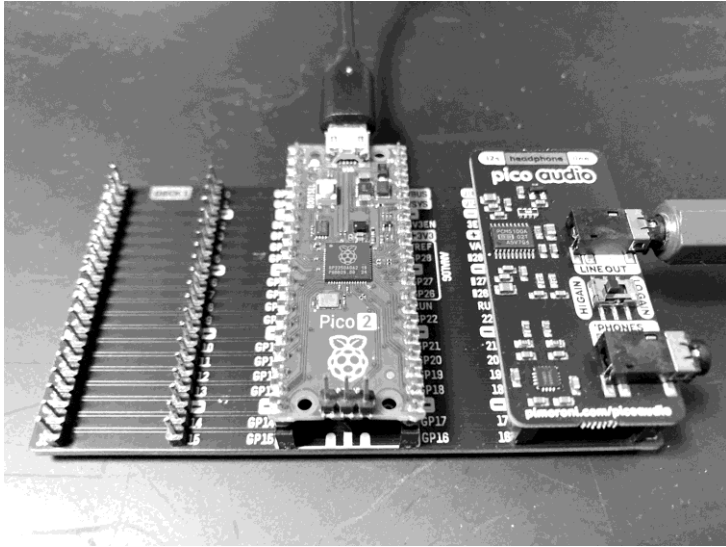


Image above: Recording result of a pitch-sweeping hard-synced square wave

- PRA32-U2 is a free 4-voice polyphonic synthesizer for Raspberry Pi Pico 2/RP2350
 - <https://risgk.github.io/#digital-synth-pra32-u2>
- PRA32-U2 provides hard-synced square and pulse waves (25%, 12.5%) over a -1 to +3 octave range, along with various binary-state waveforms



References

- [1] Daichi, シンセプログラミング, 2006.
 - <https://daichilab.sakura.ne.jp/synthprog/>
- [2] Joe Timoney, Victor Lazzarini, Matthew Hodgkinson, Jari Kleimola, Jussi Pekonen, and Vesa Välimäki, "Virtual analog oscillator hard synchronization: Fourier series and efficient implementation," Proc. of the 15th Int. Conference on Digital Audio Effects (DAFx-12), York, UK, September 17-21, 2012.
 - https://www.dafx12.york.ac.uk/papers/dafx12_submission_37.pdf

